NATIONAL EXAMINATIONS DECEMBER 2013

04-BS-5 ADVANCED MATHEMATICS

3 Hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumption made.

2. Candidates may use one of the approved Casio or Sharp calculators. This is a Closed Book Exam. However, candidates are permitted to bring **ONE** aid sheet (8.5"x11") written on both sides.

3. Any five (5) questions constitute a complete paper. Only the first five answers as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme

1. 20 marks
2. 20 marks
3. (a) 5 marks ; (b) 9 marks ; (c) 3 marks ; (d) 3 marks
4. (A) 12 marks ; (B) 8 marks
5. (A) 8 marks; (B) (i) 6 marks; (ii) 6 marks
6. 20 marks
7. (a) 10 marks ; (b) 10 marks

1.Consider the following differential equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+12y=0$$

Find two linearly independent solutions about the ordinary point x=0.

2. Find the Fourier series expansion of the periodic function f(x) of period p=2π.

$$f(x)=\begin{cases} -\pi & -\pi \le x < 0 \\ x & 0 \le x \le \pi \end{cases}$$

3.Consider the following function where a is a positive constant

$$\text{f(x)}=\begin{cases} \frac{1}{4a}\exp(x/a) & x<0 \\ \frac{1}{4a}\exp(-x/a) & x \ge 0 \end{cases}$$

(a) Compute the area bounded by f(x) and the x-axis. Graph f(x) against x for $a=0.5$ and $a=2.0$

(b) Find the Fourier transform F(ω) of f(x).

(c) Graph F(ω) against ω for the same two values of a mentioned in (a).

(d) Explain what happens to f(x) and F(ω) when a tends to infinity.

Note: $$F(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\exp(-i\omega x)dx$$

4.(A) Set up Newton's divided difference formula for the data tabulated below and derive the polynomial of highest possible degree.

x	−5	−3	−2	0	2	3	4
F(x)	0	6	−6	−30	−14	24	90

4.(B) Use the formulas supplied below to find the approximate value of the first, second, third and fourth derivative of the function f(x) tabulated below at $x_0 = -2$. Let h=2.

$$f'(x_0) \approx \frac{1}{12h}\left[-25f(x_0)+48f(x_0+h)-36f(x_0+2h)+16f(x_0+3h)-3f(x_0+4h)\right]$$

$$f''(x_0) \approx \frac{1}{12h^2}\left[35f(x_0)-104f(x_0+h)+114f(x_0+2h)-56f(x_0+3h)+11f(x_0+4h)\right]$$

$$f'''(x_0) \approx \frac{1}{2h^3}\left[-5f(x_0)+18f(x_0+h)-24f(x_0+2h)+14f(x_0+3h)-3f(x_0+4h)\right]$$

$$f''''(x_0) \approx \frac{1}{h^4}\left[f(x_0)-4f(x_0+h)+6f(x_0+2h)-4f(x_0+3h)+f(x_0+4h)\right]$$

x	−6	−4	−2	0	2	4	6
f(x)	−30	10	−6	−30	−14	90	330

5.(A) The equation $x^2 - 2 - \sin^2 x = 0$ has a root in the neighbourhood of $x_0 = 1.8$. Use Newton's method three times to find a better approximation to this root. (Note: Carry eight digits in your calculations)

5.(B) (i) One of the two points of intersection of the curves $y = \ln(1 + x^2)$ and $y = x^2 - 3x - 4$ lies between a=4.5 and b=4.6. Use the method of bisection five times to find a better approximation to this root. (ii) Consider now the equation $\ln(1+ x^2) - x^2 + 3x + 4 = 0$. This equation can be written in the form $x = \{\ln(1 + x^2) + 3x + 4\}/x$. Use fixed point iteration three times to find a better approximation to the root. Start with the last value you obtained in (i). (Note: Carry seven digits in your calculations).

6.The following results were obtained in a certain experiment:

x	1	2	3	4	5	6	7	8	9
y	12	17	21	24	27	25	23	20	16

Use Romberg's algorithm to obtain an approximation of the area bounded by the unknown curve represented by the table and the lines x = 1, x = 9 and y = 0.

Note: The Romberg algorithm produces a triangular array of numbers, all of which are numerical estimates of the definite integral $\int_a^b f(x)dx$. The array is denoted by the following notation:

R(1,1)
R(2,1) R(2,2)
R(3,1) R(3,2) R(3,3)
R(4,1) R(4,2) R(4,3) R(4,4)

where

$$R(1,1) = \frac{H_1}{2}[f(a) + f(b)]$$

$$R(k,1) = \frac{1}{2}\left[R(k-1,1) + H_{k-1}\sum_{n=1}^{n=2^{k-2}} f(a + (2n-1)H_k)\right]; \qquad H_k = \frac{b-a}{2^{k-1}}$$

$$R(k,j) = R(k,j-1) + \frac{R(k,j-1) - R(k-1,j-1)}{4^{j-1} - 1}$$

7.(a) The matrix $A = \begin{bmatrix} 2 & 4 & -3 \\ -4 & -10 & 11 \\ 6 & 2 & 10 \end{bmatrix}$ can be written as the product of the lower triangular matrix $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$ and the upper triangular matrix $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$. Find L and U.

7. (b) Use the results obtained in (a) to solve the following system of three linear equations:

$$
\begin{aligned}
2x_1 + 4x_2 - 3x_3 &= -16 \\
-4x_1 - 10x_2 + 11x_3 &= 52 \\
-6x_1 + 2x_2 + 10x_3 &= 34
\end{aligned}
$$